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Neutrino Masses in the Effective Rank-5 Subgroups of E_6 II: Supersymmetric Case

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Abstract

We present a complete analysis of the neutral fermion sector of supersymmetric E_6-inspired low energy models containing an extra $SU(2)$, concentrating on the Alternate Left-Right and Inert models. We show that the R-parity conserving scenario always exhibits a large Dirac mass for ν_L with maximal mixing with an isosinglet neutrino, and that R-parity violating scenarios do not change the picture other than allowing further mixing with another isosinglet. In order to recover Standard Model phenomenology, additional assumptions in the form of discrete symmetries and/or new interactions are needed. We introduce and investigate Discrete Symmetry method and Higher Dimensional Operators as mechanisms for solving the neutrino mass and mixing problems in these models.

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I. INTRODUCTION

Supersymmetric grand unified theories are among the most attractive scenarios for physics beyond the Standard Model. They are well motivated by superstring theories which may lead to a consistent theory of all interactions. Of these, SU(5) models have been studied extensively. The minimal SU(5) models predicted too large a decay rate for the proton and had to be modified. More recently, doubts have been raised about the validity of even modified SU(5) models, due to the discovery of solar [1] and atmospheric [2] neutrino oscillations. Small neutrino masses can be explained most elegantly through the seesaw mechanism, which requires the presence of a right-handed neutrino, a particle not naturally present in the spectrum of SU(5). Though scenarios with an extended neutrino sector exist in SU(5), it is worthwhile investigating Grand Unified Theories (GUTs) which naturally contain the right-handed neutrino. Experimental data from the Los Alamos Liquid Scintillation Detector (LSND) requires neutrino mass square splittings [3] which are in serious disagreement with other results unless one or more neutrinos are added and are "sterile" [4]. Such scenarios have been studied extensively [5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. Since the mixing of sterile and active neutrinos affects the interpretation of results from solar and atmospheric neutrino experiments, limits have been set on such mixings.

Sterile neutrinos can occur naturally in supersymmetric GUTs, which often predict the existence of exotic fermions. Of these, superstring-inspired E_6 is one of the most attractive choices. E_6 is the next anomaly-free choice group after $SO(10)$. It is based on an exceptional Lie group with complex representations, where each generation of fermions can be placed in the **27**-plet representation.

The E_6 spectrum contains several neutral exotic fermions, some which could be interpreted as sterile neutrinos. The precise details of mass generation and mixing with the active neutrinos would depend the particular subgroup of E_6 considered. There are many phenomenologically acceptable low energy models which arise from E_6. In this work we concentrate on rank-5 subgroups, which always break to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\eta$ [15, 16], and which contain an extra $SU(2)$ symmetry in addition to the MSSM symmetry. These intermediate subgroups give rise to the usual Supersymmetric Left-Right Model (LR-SUSY) [17], the Alternative Left-Right Supersymmetric model (ALR) [18] and the Supersymmetric Inert model [20]. In a previous work we have shown that, contrary to expectation,

and despite a rich exotic sector in the neutral fermionic sector containing three extra states, the non-supersymmetric version of these models did not provide neutrino masses and mixing consistent with neutrino experiments [19]. In this paper we analyze masses and mixings of neutrinos in supersymmetric E_6 inspired models. Before we proceed, we summarize our previous results.

In the non-supersymmetric version of either the ALR or the Inert model (and the discussion is the same for the LR model), the lightest state in the neutral fermion sector contains only $SU(2)_L$ singlets, which do not interact with SM particles. Additionally, the models predict two more light neutrino states with masses of the order of the up quark mass. These are phenomenologically unacceptable. In order to cure these problems, additional symmetries and/or new interactions are needed. In the simplest such non-minimal scenario, the Discrete Symmetry method requires imposing one extra discrete symmetry only. The aim is to eliminate the tree-level Dirac mass in the Lagrangian, thus generating radiative masses only for neutrinos. This method requires an extra $SU(2)_L$ Higgs doublet with vanishing vacuum expectation value (vev). It cures the Dirac neutrino mass problem, but predicts large mixing between active and sterile states.

The second method, the Higher Dimensional Operators (HDO) method, requires additional Higgs fields from the $\overline{\mathbf{27}}$-plet of E_6 and the existence of some intermediate scale. Higher dimensional (dimension-5) operators induce interactions which are suppressed by one power of the compactification scale. This method solves the neutrino mass problems but does not predict any sterile component(s) in the lightest neutrino state, which is now an admixture of ν_L and N_L, an exotic ($SU(2)_L$ doublet) particle. The effect of this mixing is to lower the electron neutrino coupling to the electron and the W_L. There exist similar reductions for the muon and tau couplings. Furthermore, when the reduction is different for each generation, this will violate lepton universality. See Ref. [19] for details.

The last method introduced is the Additional Neutral Fermion (ANF), which requires the existence of both new particles and new discrete symmetries. The additional interactions are of the type $\mathbf{27} \cdot \overline{\mathbf{27}} \cdot \mathbf{1}$, which further require additional Higgs doublets from the $\mathbf{27} + \overline{\mathbf{27}}$ representation. In order not to alter existing couplings, the vev's of the new fields must be chosen suitably, and an additional Z_2 symmetry is needed. In these circumstances, we obtain two light states with an active neutrino part of the form predicted by the HDO method, but mixed with a sterile flavor state. The mixing is completely arbitrary. Extended to three

generations, the model contains two structures, $\mathbf{2}+\mathbf{2}$ and $\mathbf{3}+\mathbf{1}$, or, if the above mixing is sizable only for one generation, the $\mathbf{2}+\mathbf{2}$ structure arises naturally. Otherwise, more realistically, including three generations for each exotic neutral fermion, we obtain a $\mathbf{3}+\mathbf{3}$ structure.

This paper is organized as follows. We discuss supersymmetrized versions of the Alternative Left-Right and Inert models in Section II. In Section III we analyze neutrino masses and mixings in the ALR and Inert models within the R-parity conserving scenario. Both of these models suffer from predicting too large a Dirac mass for the active neutrinos. The possible mixing between $R=+1$ and $R=-1$ sectors through soft R-parity violating terms is discussed for each models separately in Section IV and in an Appendix. All possible hierarchies among the parameters exhibit the feature that the physically relevant state still has too large a mass and the lightest state is fully sterile. So, in Section V, we suggest mechanisms for going beyond the minimal content of the models in order to rectify this problem. We conclude and summarize our results in Section VI.

II. DESCRIPTION OF THE MODELS

The details of the models are given in our earlier work [19]. Here we would like to summarize our previous results and concentrate on the Higgs sectors of the SUSY models where the difference occurs with respect to their non-SUSY versions.

Under the maximal subgroup $SU(3)_C \otimes SU(3)_L \otimes SU(3)_H$ of E_6, the $\mathbf{27}$ dimensional representation of E_6 branches into

$$\begin{aligned} \mathbf{27} &= (\mathbf{3^c},\mathbf{3},\mathbf{1}) + (\mathbf{\bar{3}^c},\mathbf{1},\mathbf{\bar{3}}) + (\mathbf{1^c},\mathbf{\bar{3}},\mathbf{3}) \\ &= \quad q \quad + \quad \bar{q} \quad + \quad l\,, \end{aligned} \tag{2.1}$$

where

$$q = \begin{pmatrix} u \\ d \\ h \end{pmatrix}_L, \quad \bar{q} = (u^c\ d^c\ h^c)_L\,, \quad l = \begin{pmatrix} E^c & N & \nu \\ N^c & E & e \\ e^c & \nu^c & S^c \end{pmatrix}_L. \tag{2.2}$$

Here $SU(3)_H$ operates horizontally. There are three different ways to break $SU(3)_H$ into $SU(2)_H \otimes U(1)_{Y_H}$. When the first and the second columns form a $SU(2)_H$ doublet, the so-called Left-Right (LR) symmetric model is obtained. Its alternative version is when the first

TABLE I: The quantum numbers of fermions in **27** of E_6 at $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_{V=Y_L+Y_{R'}}$ and $SU(3)_C \otimes SU(2)_L \otimes SU(2)_I \otimes U(1)_Y$ levels.

state	I_{3L}	$I_{3R'}$	I_{3I}	$V/2$	$Y/2$	Q_{em}
u_L	1/2	0	0	1/6	1/6	2/3
u_L^c	0	-1/2	0	-1/6	-2/3	-2/3
d_L	-1/2	0	0	1/6	1/6	-1/3
d_L^c	0	0	-1/2	1/3	1/3	1/3
h_L	0	0	0	-1/3	-1/3	-1/3
h_L^c	0	1/2	1/2	-1/6	1/3	1/3
e_L	-1/2	-1/2	-1/2	0	-1/2	-1
e_L^c	0	1/2	0	1/2	1	1
E_L	-1/2	0	1/2	-1/2	-1/2	-1
E_L^c	1/2	1/2	0	0	1/2	1
ν_L	1/2	-1/2	-1/2	0	-1/2	0
ν_L^c	0	0	1/2	0	0	0
N_L	1/2	0	1/2	-1/2	-1/2	0
N_L^c	-1/2	1/2	0	0	1/2	0
S_L^c	0	-1/2	-1/2	1/2	0	0

and the third columns form a doublet, which is the Alternative Left-Right (ALR) symmetric model. The last combination is when the second and the third columns combine to form a doublet and the Inert model is obtained. See Ref. [19] for more details.

In the LR and ALR models, both $SU(2)_H$ and $U(1)_{Y_H}$ contribute to the electromagnetic charge Q_{em}. In the Inert model, however, $SU(2)_H$ does not contribute to Q_{em}, which leads to neutral gauge bosons and a very different phenomenology [20]. We will use the notation $H = R, R', I; Y_H = Y_{R,R',I}$ for the LR, ALR and Inert groups, respectively. We consider their rank-5 versions whose gauge groups are $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_V$, $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_V$, and $SU(3)_C \otimes SU(2)_L \otimes SU(2)_I \otimes U(1)_Y$ for LR, ALR and Inert cases, respectively. The quantum numbers of the particles in the ALR and Inert models are given in Table I.

The Higgs sector of E_6 in the SUSY scenario differs from the non-SUSY case. Since SUSY requires doubling the number of particles, there exist many scalar fields, some of which may be taken as the Higgs bosons required for symmetry breaking. In fact, there are two ways to proceed [15]. One could assign the Higgs fields to the same **27** (or to a $\overline{\mathbf{27}}$) as the usual fermions and then some of the superpartners of the fermions can play the role of the Higgs fields. Or, it is possible to assign them to different **27** representations than the fermions, and the Higgs fields are introduced as additional scalars. The latter is less economical than the former and very similar to the non-SUSY case which was discussed in the earlier paper [19]. So we choose to work in the former framework. In fact, our approach is to choose as many Higgs bosons as possible among the superpartners of lepton fields and consider other scalars from different **27**'s only if necessary.

To analyze the Higgs sector further, we need to write the most general R-parity conserving renormalizable superpotential invariant under the Standard Model gauge group [15]

$$
\begin{aligned}
W &= W_1 + W_2 + W_3 + W_4 \,, \\
W_1 &= \lambda_1 Q u_L^c H^c + \lambda_2 Q d_L^c H + \lambda_3 L e_L^c H + \lambda_4 H H^c S_L^c + \lambda_5 h_L h_L^c S_L^c \,, \\
W_2 &= \lambda_6 h_L u_L^c e_L^c + \lambda_7 L Q h_L^c + \lambda_8 d_L^c \nu_L^c h_L \,, \\
W_3 &= \lambda_9 Q Q h_L + \lambda_{10} h_L^c u_L^c d_L^c \,, \\
W_4 &= \lambda_{11} L H^c \nu_L^c \,,
\end{aligned}
\tag{2.3}
$$

where the following notation is used:

$$
Q = \begin{pmatrix} u \\ d \end{pmatrix}_L , \; L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L , \; H = \begin{pmatrix} N \\ E \end{pmatrix}_L , \; H^c = \begin{pmatrix} E^c \\ N^c \end{pmatrix}_L . \tag{2.4}
$$

In each term in W, one of three fields corresponds to a scalar field and thus each term represents three different Yukawa interactions. We later discuss the ALR and Inert models by further imposing $SU(2)$ symmetries on the superpotential W, which reduces the number of independent Yukawa couplings. Now, before choosing the Higgs fields to be superpartners of the fermions, we first must determine the baryon (B) and lepton (L) numbers (and R parity) of the exotic fields (the ones in Table I other than the Standard Model fields). To get consistent B and L assignments, not all of the terms in W can exist simultaneously. Possibilities can be classified with respect to the (B, L, R) assignments of the fields h_L and ν_L^c. The existence of the W_2-term requires $(B, L, R)_{h_L} = (1/3, 1, -1)$ (h_L is a leptoquark),

the W_3-term requires $(B,L,R)_{h_L} = (-2/3, 0, -1)$ (h_L is a diquark). Clearly, both the W_2 and W_3 terms can not exist simultaneously without violating baryon and lepton numbers. If one wished to treat h_L as an ordinary quark[1], $(B,L,R)_{h_L} = (1/3, 0, +1)$, then both W_2 and W_3 would be eliminated. For the ν_L^c field there are two possibilities; $(B,L,R)_{\nu_L^c} = (0,1,+1)$ or $(B,L,R)_{\nu_L^c} = (0,0,-1)$. Unlike the former, the latter assignment allows a non-zero vev for the superpartner of ν_L^c, $\tilde{\nu}_L^c$, without violating lepton number. But this non-zero vev is needed only for rank-6 models. In our discussion, we are free to choose either way, and we follow the former since rank-5 models are considered. In addition, inducing a negative mass for $\tilde{\nu}_L^c$ via the renormalization group may not always be possible due to the necessity of large Yukawa couplings [15, 21]. In the rest of our discussion we take h_L as leptoquark ($W_3 = 0$), for reasons to be discussed shortly and call for the usual assignment to ν_L^c.

Now, the ALR and Inert models are defined as follows:

1. <u>The ALR Model:</u>

   The $SU(2)$ symmetry (the so-called $SU(2)_{R'}$), $H^c \Leftrightarrow L$, $u_L^c \Leftrightarrow h_L^c$, $e_L^c \Leftrightarrow S_L^c$, imposed on the superpotential W, gives rise to the effective rank-5 version of ALR model and sets the following relations among λ's: $\lambda_1 = \lambda_7$, $\lambda_3 = \lambda_4$, $\lambda_5 = \lambda_6$. Hence, by modifying the Yukawa couplings accordingly, the superpotential for the ALR model is written in a more compact form

$$
\begin{aligned}
W_{\mathrm{ALR}} &= -\lambda_1 L_A^c F_A H + \frac{\lambda_2}{2} F_A F_A \nu_L^c + \lambda_3 Q F_A X_A^c + \lambda_4 d_L^c Q H \\
&\quad + \lambda_5 h_L X_A^c L_A^c + \lambda_6 h_L d_L^c \nu_L^c \\
&= \lambda_1 \left(e_L e_L^c N_L - N_L N_L^c S_L^c + E_L E_L^c S_L^c - e_L^c E_L \nu_L \right) + \lambda_2 \left(\nu_L \nu_L^c N_L^c - e_L E_L^c \nu_L^c \right) \\
&\quad + \lambda_3 \left(u_L u_L^c N_L^c - d_L u_L^c E_L^c + d_L h_L^c \nu_L - u_L h_L^c e_L \right) + \lambda_4 \left(-d_L d_L^c N_L + d_L^c u_L E_L \right) \\
&\quad + \lambda_5 \left(-h_L h_L^c S_L^c + h_L u_L^c e_L^c \right) + \lambda_6 h_L d_L^c \nu_L^c, \qquad (2.5)
\end{aligned}
$$

   where the following definitions are used in the first form of the above equation:

$$
F_A \equiv \left(H^c\ L\right)_L = \begin{pmatrix} E_L^c & \nu_L \\ N_L^c & e_L \end{pmatrix}, \; L_A^c = \left(e^c\ S^c\right)_L, \; X_A^c = \left(h^c\ u^c\right)_L . \qquad (2.6)
$$

[1] In fact, no direct constraint comes from the W_1-term, but all other considerations lead to a stable h_L which is phenomenologically problematic. See Ref. [15] for details.

When the usual assignments of the Standard Model fields are taken, the baryon and the lepton numbers (with R-parity) of the exotics are $(B,L,R)_H = (0,0,-1), (B,L,R)_{S_L^c} = (0,0,-1)$. As discussed above, of the two possible assignments for ν_L^c in ALR model, the choice $(B,L,R)_{\nu_L^c} = (0,-1,1)$ is adopted. We also choose h_L as a leptoquark $(B,L,R)_{h_L} = (1/3,1,-1)$. It should be noted that even though there are three possibilities for the assignments of the h_L quantum numbers (leptoquark, di-quark or quark) at the E_6 level, h_L is forced to be a leptoquark in both the ALR and Inert models. This is simply because of the fact that the $SU(2)$ symmetries which convert $e_L^c \Leftrightarrow S_L^c$ and $\nu_L^c \Leftrightarrow S_L^c$ for the ALR and Inert models respectively would otherwise be broken.

Thus, the superpartners of N_L, N_L^c and S_L^c $(\tilde{N}_L, \tilde{N}_L^c, \tilde{S}_L^c)$ are possible candidates which can play the role of the neutral Higgs fields. So the Higgs sector of the ALR model that we adopt is

$$H_1 = \begin{pmatrix} \phi_1^+ & \tilde{S}_L^c \end{pmatrix}, \quad H_2 = \begin{pmatrix} \tilde{N}_L \\ \tilde{E}_L \end{pmatrix}, \quad H_3 = \begin{pmatrix} \tilde{E}_L^c & \tilde{\nu}_L \\ \tilde{N}_L^c & \tilde{e}_L \end{pmatrix}, \quad H_S = \phi_S^0. \tag{2.7}$$

Here the non-zero vev's are $\langle \tilde{S}_L^c \rangle = N_1$, $\langle \tilde{N}_L \rangle = v_1$, $\langle \tilde{N}_L^c \rangle = v_3$, $\langle \phi_S^0 \rangle = N_2$. In principle, one could have a non-zero vev for $\tilde{\nu}_L$, but this would violate lepton number (the neutrino, ν_L, would get a large Majorana mass, through $\tilde{Z}_L$ exchange, of order $\langle \tilde{\nu}_L \rangle^2 / M_{\tilde{Z}_L}$). We will assume that $\tilde{\nu}_L$ has no vev for the moment, but will mention the effects of the other possibility later. This other possibility could be acceptable if, for example, the $\tilde{\nu}_L$ comes from a different **27** than the Standard model fermions. One can choose ϕ_S^0 to be a singlet or $\tilde{\nu}_L^c$. In the latter case, either N_2 needs to be zero or $(B,L,R)_{\nu_L^c} = (0,0,-1)$ should be adopted. Here, we keep our discussion general.

2. The Inert Model:

In this case, the $SU(2)$ symmetry (the so-called $SU(2)_I$), $H \Leftrightarrow L$, $d_L^c \Leftrightarrow h_L^c$, $\nu_L^c \Leftrightarrow S_L^c$, imposed on the superpotential W of Eq. (2.3) leads to the effective rank-5 version of the Inert model. Thus, the following relations among the Yukawa couplings hold: $\lambda_2 = \lambda_7$, $\lambda_4 = \lambda_{11}$, $\lambda_5 = \lambda_8$. Similar to the ALR model case, the superpotential of the Inert model is expressed as

$$W_{\rm Inert} = \lambda_1' L_I^c F_I H^c - \frac{\lambda_2'}{2} F_I F_I e_L^c + \lambda_3' h_L X_I^c L_I^c + \lambda_4' h_L u_L^c e_L^c + \lambda_5' Q u^c H^c + \lambda_6' Q X_I^c F_I$$

$$
\begin{aligned}
&= \lambda_1' \left(\nu_L \nu_L^c N_L^c + N_L N_L^c S_L^c + E_L E_L^c S_L^c + e_L \nu_L^c E_L^c\right) \\
&\quad + \lambda_2' \left(e_L e_L^c N_L + \nu_L \nu_L^c N_L^c + e_L^c E_L \nu_L\right) + \lambda_3' \left(h_L h_L^c S_L^c + h_L d_L^c \nu_L^c\right) + \lambda_4' h_L u_L^c e_L^c \\
&\quad + \lambda_5' \left(u_L u_L^c N_L^c + d_L u_L^c E_L^c\right) \\
&\quad + \lambda_6' \left(d_L d_L^c N_L + d_L h_L^c \nu_L + u_L d_L^c E_L + u_L h_L^c e_L\right) , \qquad (2.8)
\end{aligned}
$$

where the following definitions are used

$$
F_I \equiv (H \ L)_L = \begin{pmatrix} N_L & \nu_L \\ E_L & e_L \end{pmatrix} , \ L_I^c = \left(\nu^c \ S^c\right)_L , \ X_I^c = \left(h^c \ d^c\right)_L . \qquad (2.9)
$$

The baryon and lepton number assignments for exotics are similar to the ALR model. $(B, L, R)_H = (0, 0, -1)$ and $(B, L, R)_{S_L^c} = (0, 0, -1)$ apply and h_L is also considered as leptoquark as discussed above. Unlike the ALR case, ν_L^c is forced to have assignments $(B, L, R)_{\nu_L^c} = (0, -1, 1)$. Thus, a vev for $\tilde{\nu}_L^c$ is not allowed unless lepton flavor violating interactions are included. From these considerations we choose the Higgs content of the model as follows

$$
H_D = \begin{pmatrix} \phi_S^0 & \tilde{S}_L^c \end{pmatrix} , \quad H_2 = \begin{pmatrix} \tilde{E}_L^c \\ \tilde{N}_L^c \end{pmatrix} , \quad H_3 = \begin{pmatrix} \tilde{N}_L & \tilde{\nu}_L \\ \tilde{E}_L & \tilde{e}_L \end{pmatrix} , \quad H_S = \phi_1^+, \qquad (2.10)
$$

with the following vev's, $\langle \tilde{S}_L^c \rangle = N_1$, $\langle \tilde{N}_L \rangle = v_1$, $\langle \tilde{N}_L^c \rangle = v_3$, $\langle \phi_S^0 \rangle = N_2$. Here the $SU(2)_I$ doublet H_D is electrically neutral while ϕ_S^0 is possibly taken as $\tilde{\nu}_L^c$ with zero vev. As before, we assume that $\langle \tilde{\nu}_L \rangle = 0$, but will consider the alternative possibility later.

III. NEUTRINOS IN THE ALR AND INERT MODELS

In this section we analyze the neutral fermion sectors of both the ALR and Inert models[2] by using the superpotentials given in Eqs. (2.5) and (2.8). The superpotentials only describe $(\mathbf{27})^3$ type interactions. Without considering any more particles or new interactions, there exists a 5×5 "neutrino" mass matrix for each generation. From R-parity considerations

[2] Since this paper is concentrating on neutrinos, we will not discuss mixing between light and heavy fields in the charged lepton or quark sectors. Such mixing can have a wide range of interesting phenomenological effects, see Ref. [22] for a detailed discussion and a list of references.

this 5×5 matrix splits into 2×2 and 3×3 submatrices. From Eqs. (2.5) and (2.8), the $R = +1$ neutral fermion sector spanned by $(\nu_L,\, \nu_L^c)$ becomes

$$\mathcal{M}^{R=+1} = \begin{pmatrix} 0 & m_{\nu\nu^c} \\ m_{\nu\nu^c} & 0 \end{pmatrix}, \tag{3.1}$$

where $m_{\nu\nu^c} = \lambda_2 \langle \tilde{N}_L^c \rangle = \lambda_2 v_3$ in the ALR and $m'_{\nu\nu^c} = \lambda'_1 v_3$ in the Inert model. Clearly, the ordinary neutrinos have a Dirac mass $m_{\nu\nu^c}$ which is of the order of the up quark mass in both models and the physical state is formed by the maximal mixing of ν_L and ν_L^c. Either an unnatural fine tuning for the Yukawa couplings is needed, or we must introduce a large Majorana mass for ν_L^c which renders a small Majorana mass for ν_L through the canonical seesaw mechanism [23]. Another possibility is to generate a small Dirac one-loop mass by eliminating the tree level mass term. The possibilities will be discussed shortly.

The $R = -1$ sector is composed of 3 neutral leptons, N_L, N_L^c, S_L^c, and 3 neutral gauge fermions corresponding to two the $SU(2)$'s and one $U(1)$ group. For simplicity, we assume that the gauge fermions get large Majorana mass terms from soft-supersymmetry breaking and decouple. The 3×3 Majorana mass matrices in the (N_L, N_L^c, S_L^c) basis become

$$\mathcal{M}_{\mathrm{ALR}}^{R=-1} = \begin{pmatrix} 0 & -m_{EE^c} & -\lambda_1 v_3 \\ -m_{EE^c} & 0 & -m_{ee^c} \\ -\lambda_1 v_3 & -m_{ee^c} & 0 \end{pmatrix}, \; \mathcal{M}_{\mathrm{I}}^{R=-1} = \begin{pmatrix} 0 & m'_{EE^c} & m'_{\nu\nu^c} \\ m'_{EE^c} & 0 & m'_{ee^c} \\ m'_{\nu\nu^c} & m'_{ee^c} & 0 \end{pmatrix}, \tag{3.2}$$

where $m_{EE^c} = \lambda_1 N_1$, $m_{ee^c} = \lambda_1 v_1$, $m'_{EE^c} = \lambda'_1 N_2$, $m'_{ee^c} = \lambda'_1 v_1$. Here $\mathcal{M}_{\mathrm{ALR}}(\mathcal{M}_{\mathrm{I}})$ is the $R = -1$ mass matrix in the ALR (Inert) model. Diagonalization of the above matrix for the ALR case leads to the states and masses

$$\begin{aligned} |\nu_{1,2}\rangle_{\mathrm{ALR}} &\simeq \frac{1}{\sqrt{2}} \left(|N_L\rangle \pm |N_L^c\rangle \right), \quad M_{1,2}^H \simeq \pm m_{EE^c}, \\ |\nu_3\rangle_{\mathrm{ALR}} &\simeq |S_L^c\rangle, \qquad\qquad\qquad\quad M_3^L \simeq 2\lambda_1 v_3 m_{ee^c}/m_{EE^c}, \end{aligned} \tag{3.3}$$

under the assumption $m_{ee^c} \sim \lambda_1 v_3 \ll M_{EE^c}$. Here the superscripts H and L stand for the heavy and light states, and λ_1 can further be express as m_{EE^c}/N_1. Similar results apply to the Inert model, where the states are the same with masses $M'^H_{1,2} = \pm m'_{EE^c}$ and $M'^L_3 = -2m'_{\nu\nu^c} m'_{ee^c}/m'_{EE^c}$, respectively. Clearly, there are two heavy ($|\nu_{1,2}\rangle_{\mathrm{ALR(I)}}$) states and one light ($|\nu_3\rangle_{\mathrm{ALR(I)}}$) state. The light (mainly sterile) state does not however mix with active neutrinos unless R-parity is broken. In the non-SUSY framework [19], without introducing

further symmetries or interactions the lightest state is formed by ν_L^c and S_L^c . In the SUSY scenario, it is still possible to mix $R = +1$ and $R = -1$ sectors by including soft-symmetry breaking terms [24]. This will be discussed in the next section.

At this stage, one can take $(\tilde{\nu}_L\ \tilde{e}_L)$ with non-zero vev for $\tilde{\nu}_L$ without changing the above results. However, for the case $\phi_S^0 = \tilde{\nu}_L^c$ with zero vev, the results are modified for the Inert model but remain unchanged for ALR. Then, the $R = -1$ sector of the Inert model has the following states and masses

$$\begin{aligned}
|\nu'_{1,2}\rangle_{\rm I} &\xrightarrow{\langle\tilde{\nu}_L^c\rangle=0} \frac{1}{\sqrt{2}}\left(\frac{m'_{\nu\nu^c}}{M_1'^H}|N_L\rangle + \frac{m'_{ee^c}}{M_1'^H}|N_L^c\rangle \pm |S_L^c\rangle\right), & M_{1,2}'^H &\xrightarrow{\langle\tilde{\nu}_L^c\rangle=0} \pm\sqrt{m'^2_{\nu\nu^c}+m'^2_{ee^c}}, \\
|\nu'_3\rangle_{\rm I} &\xrightarrow{\langle\tilde{\nu}_L^c\rangle=0} \frac{1}{M_1'^H}\left(-m'_{ee^c}|N_L\rangle + m'_{\nu\nu^c}|N_L^c\rangle\right), & M_3'^L &\xrightarrow{\langle\tilde{\nu}_L^c\rangle=0} 0,
\end{aligned} \tag{3.4}$$

For the case $\phi_S^0 = \tilde{\nu}_L^c$ with non-zero vev (that is, when $(B, L, R)_{\nu_L^c} = (0, 0, -1)$ is adopted), the $R = -1$ sector of the ALR model would be a 4×4 matrix. This possibility is solely available for the ALR model, since R-parity conservation requires λ_1' in Eq. (2.8) to vanish. In ALR, λ_2 should be eliminated by imposing some discrete symmetries in order not to break R-parity conservation. However, this also decouples ν_L^c from the 4×4 matrix and makes it massless. So, no change occurs in the 3×3 submatrix and both ν_L and ν_L^c become massless. Note that in this framework ν_L^c is no longer a Dirac conjugate pair state of the active ν_L neutrino but it is a sterile neutrino with zero lepton number. In the next section, we discuss possible mechanisms to generate small Majorana masses for active neutrinos and possible mixing between opposite R-parity sectors.

IV. GIVING MASS THROUGH R-PARITY BREAKING

The fact that the R-parity might be broken by soft terms [24] has been discussed by Ma in the context of ALR model [25]. The idea is as follows. A soft term which describes a mixing between ν_L and N_L^c can be realized by, for example, giving a vev to $\tilde{\nu}_L^c$ in the $F_AF_A\nu_L^c$ term of Eq. (2.5). It can be defined as $\mu_A(\nu_L N_L^c - e_L E_L^c)$. The presence of such mixing then induces a mixing between ν_L and the lightest state ν_3 through a small N_L^c component of ν_3. Then, the active neutrino mass matrix is enlarged from 2×2 to 3×3 and is given by, in

the basis (ν_L, ν_L^c, S_L^c) [3]

$$\mathcal{M}_{\rm ALR}^{R=+1} = \begin{pmatrix} 0 & m_{\nu\nu^c} & m_S \\ m_{\nu\nu^c} & 0 & 0 \\ m_S & 0 & M_3^L \end{pmatrix}. \tag{4.1}$$

where $m_S \equiv m_{\nu\nu^c}\mu_A/m_{EE^c}$ and $M_3^L \simeq 2\lambda_1 v_3 m_{ee^c}/m_{EE^c}$. The corresponding matrix for Inert model is $\mathcal{M}_{\rm I}^{R=+1} = \mathcal{M}_{\rm ALR}^{R=+1}\left(m_{\nu\nu^c}(M_3^L) \to m'_{\nu\nu^c}(M_3'^L), m_S \to m'_S \equiv m'_{\nu\nu^c}\mu_I/m'_{EE^c}\right)$. Here $M_3'^L$ is defined as $M_3'^L = -2m'_{\nu\nu^c}m'_{ee^c}/m'_{EE^c}$ We envisage two limiting cases, one for μ_A very small compared with m_{EE^c}; $m_S \ll |M_3^L| \ll m_{\nu\nu^c}$ (case (i)) and one for μ_A large compared with m_{EE^c}; $|M_3^L| \ll m_S \ll m_{\nu\nu^c}$ (case (ii)). A third case is possible when μ_A is comparable with m_{EE^c}; $|M_3^L| \ll m_S \sim m_{\nu\nu^c}$ (case (iii)). In the case in which all three, $|M_3^L|, m_S$ and $m_{\nu\nu^c}$ are comparable with each other, it is not possible to draw any valuable conclusion from analytic calculations. In order get sizable mixing between active and sterile neutrinos while they are lying in the correct mass range, it is required to have comparable but small Dirac and Majorana masses [33]. The main results of the above cases and each of the corresponding spectra are summarized in Table II in Appendix.

Summarizing the results from Appendix, one finds that the spectrum has two heavy and one light states. The common feature of all cases is that the lightest state is always purely sterile, mainly composed of either S_L^c or S_L^c and ν_L^c. It has a seesaw type mass as given in Appendix. The heavy states have maximal mixing between ν_L and ν_L^c with a small component of S_L^c. For case (i) ($m_S \ll |M_3^L| \ll m_{\nu\nu^c}$), the heavy states consist of only ν_L and ν_L^c. They are too heavy to be considered the physical neutrino state as $m_{\nu\nu^c}$ is at the scale of the up quark mass.

Thus so far no satisfactory pattern for neutrino masses and mixings has been established. We are still required to go beyond the minimal picture, as we will discuss in the next section.

If the $\tilde{\nu}_L^c$ comes from a different **27**, and thus can get a non zero vev, then these results are unaffected. Choosing $\phi_S^0 = \tilde{\nu}_L^c$ with zero vev, however, makes R-parity violation disappear and the S_L^c again decouples.

[3] The third entry will be represented by S_L^c since the lightest state is mainly described by S_L^c.

V. BEYOND THE MINIMAL CONTENT

As we have seen in the previous sections, the absence of the terms in W_3, which guarantees proton stability, could be a consequence of a discrete symmetry. Note that conventional R-parity is not sufficient to explain the elimination of some Yukawa couplings. For this purpose, an odd Z_2 charge to all Standard Model quarks and h_L and an even Z_2 charge to the rest of the fields can be assigned. Clearly, invariance under this Z_2 symmetry would require λ_9 and λ_{10} to be zero. Elimination of other Yukawa couplings can be achieved by imposing further symmetries. Depending on whether the neutrinos are Dirac or Majorana particles, we can proceed in two ways.

If one assumes that the neutrino is a Dirac particle, then the Dirac neutrino mass predicted directly from the superpotential should be much smaller for both models. A solution to effectively fine tune the coupling has been proposed by Branco and Geng [26, 27]. They make the model invariant under a Z_3 symmetry in addition to the Z_2 symmetry considered above (this is what we have called the Discrete Symmetry (DS) method in our earlier paper [19]). Here the Z_3 symmetry distinguishes between generations. The symmetry eliminates the tree-level Dirac mass term from the superpotential and induces a smaller one-loop mass. In Ref. [26], the discussion has been carried out at E_6 level without reference to any of its subgroups. Assuming the invariance of E_6 itself under Z_3 symmetry, the breaking of E_6 to the $SU(2)_{R'}$ or $SU(2)_I$ symmetries lead to breaking of the Z_3 symmetries. Since the ALR and Inert models are treated as different subgroups of E_6, one can introduce Z_3 invariance after the E_6 gauge symmetry is broken.

If neutrinos are considered to be Majorana particles, then generation of small Majorana masses for left-handed (active) neutrinos could be achieved by including Higher Dimensional Operators (HDO) [28]. One can show that the next available interactions in the Standard Model are dimension-5, which can be sizable if one introduces an intermediate scale around 10^{11} GeV and Higgs fields from a $\overline{\mathbf{27}}$ representation of E_6. Through the canonical seesaw mechanism, in the $R = +1$ sectors of the models, the small Majorana mass of the left-handed active neutrino is generated by having a large Majorana mass for ν_L^c. In the $R = -1$ sector, S_L^c will also get a large Majorana mass which modifies the results discussed in Section III. If one further includes the soft-terms which break R-parity, a large coupling could occur between ν_L^c and S_L^c. So, this framework can give us a picture involving sterile neutrinos,

which is promising.

As an alternative to the above methods, one can extend the minimal content of E_6 together with its Higgs sector by further considering E_6-neutral fermion and Higgs fields from the split multiplet $\mathbf{27}+\overline{\mathbf{27}}$. This Additional Neutral Fermion (ANF) method was first proposed by Mohapatra and Valle [29, 30, 31]. This way, it is possible to produce either the Dirac or Majorana neutrinos with small mass.

Among these methods, the DS is the simplest and the most attractive one as it does not require the existence of an intermediate scale or inclusion of new particles (and interactions). The ANF method is the most complex, as it requires not only presence of some discrete symmetries but also the presence of new interactions. As indicated earlier, the occurrence of sterile neutrino components in the physical states can only be possible through soft-breaking terms. We now analyze these methods in the following subsections.

A. The Discrete Symmetry Method

As discussed above, a Z_2 symmetry which assigns odd charges to $Q, u_L^c, d_L^c, h_L, h_L^c$ fields and even charges to the rest may be required to explain the absence of W_3-terms. The DS method, as we will show, also imposes a Z_3 symmetry which eliminates the tree-level $m_{\nu\nu^c}$ and makes it appear at one-loop. It is thus much smaller. Unlike the Inert case, in the ALR model M_3^L does not depend on $m_{\nu\nu^c}$. So, one-loop Dirac mass generation for neutrinos doesn't affect M_3^L and make it comparable or even bigger than the $m_{\nu\nu^c}$ generated at one-loop (say $m_{\nu\nu^c}^{1-\text{loop}}$) in ALR. The cases for ALR should thus be reconsidered under the circumstance $m_S \ll m_{\nu\nu^c}^{1-\text{loop}} \sim M_3^L$. As a result case (ii) is irrelevant and in case (iii) the hierarchy among $|M_3^L|, m_{\nu\nu^c}$, and m_S disappears. So, three of the parameters become comparable with each other and no conclusion can be extracted in this case.

- <u>One-Loop Masses in ALR:</u>

  In addition to the Z_2 symmetry, a Z_3 symmetry is needed to set λ_2 to zero. It should of course leave the Yukawa couplings λ_3 and λ_6 in Eq. (2.5) unaffected to generate one-loop neutrino mass, and λ_1, λ_4 and λ_5 to generate masses for Standard Model quarks and charged leptons and exotics. This should be the case for at least some components of these couplings in flavor space. We know that the Z_3 symmetry should distinguish

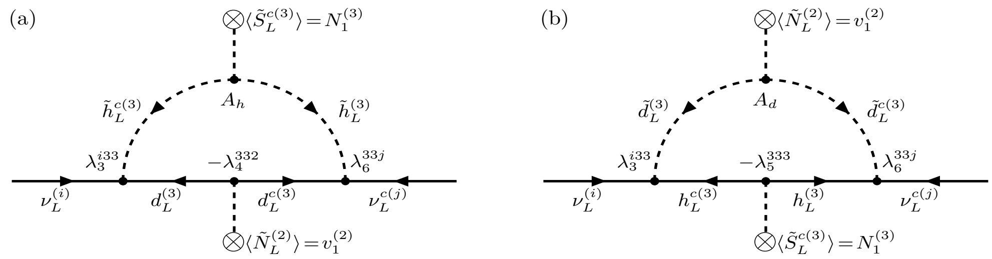

FIG. 1: The one-loop Dirac masses for $\nu_L^{(i)}\nu_L^{c(j)}$.

between generations [26]. One of such allowed symmetry charge assignments could be as follows:

$$\begin{aligned}
Z_3: \quad & [Q, d_L^c, h_L, h_L^c, L, \nu_L^c]^{(i)} \to \eta\,[Q, d_L^c, h_L, h_L^c, L, \nu_L^c]^{(i)}\ , \\
& u_L^{c(i)} \to \eta^{-1} u_L^{c(i)}\,, \\
& H^{c(1)} \to \eta^{-1} H^{c(1)}\,, \quad H^{c(2)} \to H^{c(2)}\,, \quad H^{c(3)} \to H^{c(3)}\,, \\
& H^{(1)} \to \eta^{-1} H^{(1)}\,, \qquad H^{(2)} \to \eta H^{(2)}\,, \quad H^{(3)} \to H^{(3)}\,, \\
& S_L^{c(1)} \to \eta^{-1} S_L^{c(1)}\,, \quad S_L^{c(2)} \to S_L^{c(2)}\,, \quad S_L^{c(3)} \to \eta S_L^{c(3)},
\end{aligned} \tag{5.1}$$

where $\eta^3 = 1$ and the numbers inside the parentheses represent generations. The masses for quarks, charged leptons and exotics are given as

$$\begin{aligned}
m_u &= \lambda_3^{ij3}\langle\tilde{N}^{c(3)}\rangle, \ \ m_d = \lambda_4^{ij2}\langle\tilde{N}^{(3)}\rangle, \quad m_e = \lambda_1^{ij1}\langle\tilde{N}^{(1)}\rangle, \\
m_h &= \lambda_5^{ij3}\langle\tilde{S}^{c(3)}\rangle, \ \ m_E = \lambda_1^{ij1}\langle\tilde{S}^{c(1)}\rangle, \quad m_N = \lambda_1^{ij1}\langle\tilde{S}^{c(1)}\rangle.
\end{aligned} \tag{5.2}$$

There are two one-loop diagrams as shown in Fig. 1 which contribute to the Dirac mass generation for ν_L. A trilinear scalar vertex is involved in the one-loop diagrams. We take A_h and A_d as the trilinear soft supersymmetry-breaking coefficients for $\tilde{h}_L\tilde{h}_L^c\tilde{S}_L^c$ and $\tilde{d}_L\tilde{d}_L^c\tilde{N}_L$, respectively. Only $\lambda_3, \lambda_4, \lambda_5$ and λ_6 are involved in the one-loop diagrams. If we take the mass of h_L as the typical SUSY breaking scale $m_{1/2}$, then the one-loop neutrino mass is obtained by adding two diagrams in Fig. 1

$$\begin{aligned}
m_{\nu\nu^c}^{1-\mathrm{loop}} &= m_{\nu\nu^c}^{1-\mathrm{loop(a)}} + m_{\nu\nu^c}^{1-\mathrm{loop(b)}} \\
&\simeq \frac{A_h \lambda_3^{i33}\lambda_6^{33j} m_b}{32\pi^2}
\end{aligned} \tag{5.3}$$

where we have assumed the soft supersymmetry-breaking squark masses participating in the one-loop diagrams are given as [21] $m_{\tilde{d}} \sim m_{\tilde{h}} \simeq 3m_{1/2}$ and $A_h = A_d$. In order to

obtain neutrino masses less than 0.1 eV, a bound $\lambda_3^{i33}\lambda_6^{33j} \leq 7\times 10^{-9}$ must be imposed for all i, j when A_h is taken of order one. This is not substantially smaller than typical Yukawa couplings.

As discussed above, having a Dirac mass for neutrinos less than an eV requires reconsideration of the case (i) of section IV under the new hierarchy $m_S \ll m_{\nu\nu^c}^{1-\text{loop}} \sim M_3^L$ and makes the other cases irrelevant or inconclusive. It is not possible to give some useful analytic expressions for the masses and states unless a specific relation between M_3^L and $m_{\nu\nu^c}^{1-\text{loop}}$ is set. For illustrative purposes, if $M_3^L = 3m_{\nu\nu^c}^{1-\text{loop}}$ is chosen, the physical states under the assumption $m_S \ll M_3^L = 3m_{\nu\nu^c}^{1-\text{loop}}$ become

$$\begin{aligned}
|\nu_1\rangle &\simeq \frac{1}{\sqrt{2}}\left(|\nu_L\rangle + |\nu_L^c\rangle - \frac{\xi}{2}|S_L^c\rangle\right),\ M_1 \simeq m_{\nu\nu^c}^{1-\text{loop}}, \\
|\nu_2\rangle &\simeq \frac{1}{\sqrt{2}}\left(|\nu_L\rangle - |\nu_L^c\rangle - \frac{\xi}{4}|S_L^c\rangle\right),\ M_2 \simeq -m_{\nu\nu^c}^{1-\text{loop}}, \\
|\nu_3\rangle &\simeq \frac{3\xi}{8}|\nu_L\rangle + \frac{\xi}{8}|\nu_L^c\rangle + |S_L^c\rangle, \qquad M_3 \simeq 3m_{\nu\nu^c}^{1-\text{loop}},
\end{aligned} \tag{5.4}$$

where now $\xi \equiv m_S/m_{\nu\nu^c}^{1-\text{loop}}$ is implied. We still have two states $\nu_{1,2}$ showing a bi-maximal mixing between the active ν_L neutrino and ν_L^c where as the sterile state S_L^c appears as separate. Since the masses lie in the acceptable range, it would be possible to obtain **3+2** structural models.

- <u>One-Loop Masses in Inert:</u>

There are some differences between the two models in terms of the required Z_3 charge assignments. Firstly, since both m'_S and $M_3'^L$ depend on $m'_{\nu\nu^c}$ linearly in the Inert case, the DS method doesn't change the hierarchy among them. So, there are only the three cases as discussed in section IV after the discrete symmetry is imposed. Secondly, an even Z_3 charge can be assigned to u_L^c for three generations since the λ_6 term of Eq. (2.3) is invariant under $SU(2)_I$ and it can be eliminated by Z_3 invariance without leading to any problems. Lastly and most importantly, unlike the ALR case, the λ_{11} term of Eq. (2.3) is not invariant under $SU(2)_I$ symmetry and is combined with the λ_4 term (we relabeled both as λ'_1 in Eq. (2.8)). Thus, it is not possible to eliminate the tree-level Dirac mass term (λ_{11}) for active neutrinos unless some further assumptions are made, since eliminating the λ_{11} term would also eliminate the mass terms for N_L and E_L.

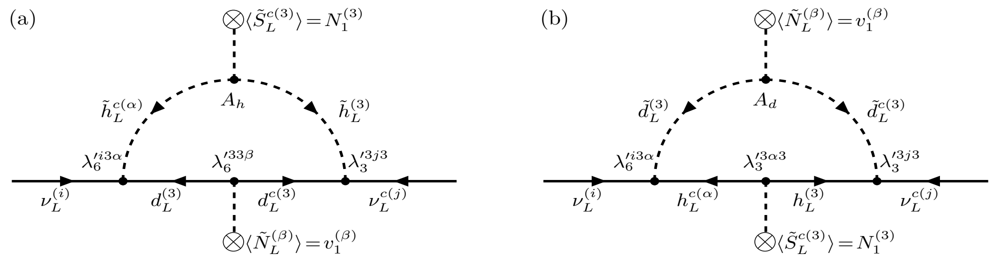

FIG. 2: The one-loop Dirac masses for $\nu_L^{(i)}\nu_L^{c(j)}$. The indices α and β run over first two generations.

The assumption needed could be to take the vev's of $\tilde{H}^c$ zero for the first two generations and giving mass to the up-quarks from $\tilde{H}^{c(3)}$ whose vev is assumed non-zero. Then the following charges could be assigned to the fields

$$
\begin{aligned}
Z_3: \quad & [Q, d_L^c, h_L, h_L^c, L, \nu_L^c]^{(i)} \to \eta\,[Q, d_L^c, h_L, h_L^c, L, \nu_L^c]^{(i)} \;, \\
& H^{c(1)} \to \eta H^{c(1)}\,, \quad H^{c(2)} \to \eta H^{c(2)}\,, \quad H^{c(3)} \to \eta^{-1} H^{c(3)}\,, \\
& H^{(1)} \;\to \eta H^{(1)}\,, \qquad H^{(2)} \to \eta H^{(2)}\,, \qquad H^{(3)} \to \eta^{-1} H^{(3)}\,, \\
& S_L^{c(1)} \to \eta^{-1} S_L^{c(1)}\,, \quad S_L^{c(2)} \to S_L^{c(2)}\,, \qquad S_L^{c(3)} \to \eta S_L^{c(3)},
\end{aligned} \tag{5.5}
$$

where

$$
\begin{aligned}
m_u &= \lambda_5'^{ij3}\langle \tilde{N}^{c(3)}\rangle, \qquad m_d = \lambda_6'^{ij\alpha}\langle \tilde{N}^{(\alpha)}\rangle, \qquad m_e = \lambda_2'^{ij3}\langle \tilde{N}^{(3)}\rangle, \\
m_h &= \lambda_3'^{ij3}\langle \tilde{S}^{c(3)}\rangle, \quad m_{E^\alpha,N^\alpha} = \lambda_1'^{ij3}\langle \tilde{S}^{c(3)}\rangle, \quad m_{E^3,N^3} = \lambda_1'^{ij1}\langle \tilde{S}^{c(1)}\rangle.
\end{aligned} \tag{5.6}
$$

Here α runs over the first and the second generations. Then, one-loop diagrams giving non-zero Dirac neutrino masses are shown in Fig. 2. Under the same assumptions as in ALR case for the calculation of the one-loop integrals, we get

$$
\begin{aligned}
m_{\nu\nu^c}'^{\text{1-loop}} &= m_{\nu\nu^c}'^{\text{1-loop(a)}} + m_{\nu\nu^c}'^{\text{1-loop(b)}} \\
&\simeq \frac{A_h \lambda_3'^{3j3}\lambda_6'^{i3\alpha} m_b}{32\pi^2}
\end{aligned} \tag{5.7}
$$

where $\alpha = 1, 2$. To get a Dirac mass $m_{\nu\nu^c}'^{\text{1-loop}} < 0.1$ eV we need to impose the bound $\lambda_3'^{3j3}\lambda_6'^{i3\alpha} < 7 \times 10^{-9}$ for $\alpha = 1, 2$. Here, unlike the ALR case, it is possible to set separate bounds on $\lambda_3'^{3\alpha 3}$ and $\lambda_6'^{33\beta}$ using fact that they give masses to the h_L and bottom quark, respectively. These bounds, however, become weaker. Having Dirac mass in the eV range makes the mostly sterile state $|\nu_3\rangle$ in all three cases too light to

be detected. The other two states in each case have the large mixing problem. As in the non-SUSY case, the DS method is only able to explain the smallness of the Dirac neutrino mass and not the mixing.

B. The Higher Dimensional Operators Method

This method adds higher dimensional interactions to the Lagrangian, which can substantially modify some of the fermion mixings. Due to the compactification scale suppression factor ($\sim 10^{19}$ GeV), it is sufficient to consider only dimension-5 interactions. The method also requires the existence of intermediate scales set by some $SU(2)_L$ singlet Higgs fields from $\overline{\mathbf{27}}$ representation of E_6. So, there will be $\tilde{S}^c_L$-like and ϕ^0_S-like scalars (H_1 and H_S) for the ALR model and an H_D-like Higgs doublet for the Inert model.[4] The vev's of these fields are written as $\langle\overline{\tilde{S}^c_L}\rangle = \Lambda_1$ and $\langle\overline{\phi_S}\rangle = \Lambda_2$. Here ϕ_S could be replaced with $\tilde{\nu}^c_L$. The dimension-5 interactions can be written as

$$\mathcal{L}^{(5)}_{\mathcal{Y}} = \frac{f}{M_c}\psi^T(\mathbf{27})\,\epsilon\,H(\overline{\mathbf{27}})\,C\,H^T(\overline{\mathbf{27}})\,\epsilon\,\psi(\mathbf{27})\,, \tag{5.8}$$

where M_c is the compactification scale ($\sim 10^{19}$ GeV) and the Higgs field $H(\mathbf{27})$ stands for both $\overline{\tilde{S}^c_L}$ and $\overline{\phi_S}$. Here C is the charge conjugation matrix defined as $C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}$ and we adopt the chiral representation and $\epsilon \equiv i\sigma_2$, where σ_2 is the Pauli matrix.

The new interactions do not sizably modify any interaction terms in the fermion mass matrices with the exception of the $\nu^c_L - S^c_L$ submatrix. The matrix in the (ν_L, ν^c_L, S^c_L) basis then becomes

$$\mathcal{M}^5 = \begin{pmatrix} 0 & m_{\nu\nu^c} & m_S \\ m_{\nu\nu^c} & K_1 & K_{12} \\ m_S & K_{12} & K_2 \end{pmatrix}, \tag{5.9}$$

where $K_i \equiv f\frac{\Lambda_i^2}{M_c}$, $i = 1, 2$ and $K_{12} \equiv 2\sqrt{K_1 K_2}$. We keep the discussion in this section general and applicable to both the ALR and Inert models unless stated. Furthermore we note that the M^L_3 term in (3,3) entry of the above matrix is negligible with respect to K_2.

---

[4] We will assume that all Higgs fields of $\mathbf{27}$ whose vev's are at the electroweak scale have corresponding $\overline{\mathbf{27}}$ Higgs fields with vev's at the same scale. All Higgs fields from the $\overline{\mathbf{27}}$ representation will have opposite quantum numbers with respect to the fields in the $\mathbf{27}$.

We now consider three cases: (a) the case in which there is only one intermediate scale of the order of 10^{12} GeV (i.e. $\Lambda_1 = \Lambda_2$), (b) the case in which Λ_1 is much smaller than Λ_2 and is of the order of 1 TeV and (c) the case in which Λ_2 is much smaller than Λ_1 and is of the order of 1 TeV.

Case (a) with $\Lambda_1 = \Lambda_2 \sim 10^{12}$ GeV makes K_1 and K_2 (and thus K_{12}) much bigger than the other entries of the matrix and of the order of 10^5 GeV. Here we are assuming the coupling constant f is of the order of unity. With big Majorana masses for ν_L^c and S_L^c, these states decouple. One light and two heavy physical states are expected. Under the assumption $m_S \ll m_{\nu\nu^c} \ll K_1 = K_2$ the states are

$$\begin{aligned} |\nu_{1,2}\rangle &\simeq \frac{1}{\sqrt{2}}\left(|\nu_L^c\rangle \pm |S_L^c\rangle\right), \qquad \overline{M_{1,2}^H}(K_1 = K_2) \simeq 3K_1, -K_1, \\ |\nu_3\rangle &\simeq |\nu_L\rangle + \frac{\zeta}{3}|\nu_L^c\rangle - \frac{2\zeta}{3}|S_L^c\rangle, \quad \overline{M_3^L}(K_1 = K_2) \simeq \frac{1}{3}\zeta^2 K_1, \end{aligned} \tag{5.10}$$

where $\zeta \equiv \frac{m_{\nu\nu^c}}{K_1}$ and is of order 10^{-8}, and the orthonormality of the states is guaranteed up to $O(\zeta)$ since ζ^2 and higher terms are not included. The ν_L^c and S_L^c components of ν_3 are shown because their mixing with ν_L is order of $\zeta \sim 10^{-8}$, which is too small to be relevant. We get a seesaw-like small Majorana mass $\overline{M_3^L}(K_1 = K_2) \simeq \frac{1}{3}\zeta^2 K_1$ for ν_L, which is around 3×10^{-3} eV. Thus, consideration of case (a) with $\Lambda_1 = \Lambda_2 \sim 10^{12}$ GeV gives rise to a single state having acceptable eV range mass with negligible active-sterile mixing. Furthermore, S_L^c and ν_L^c appear as two distinct flavor states in ν_3.

The above results show that, in order to have two light states with significant active-sterile mixing, there should be a substantial hierarchy between K_1 and K_2 in order that only one of the states $|\nu_{1,2}\rangle$ decouples, which leaves two light states. We now consider case (b) in which Λ_1 is much smaller than Λ_2. This also leads to a large ν_L Majorana mass since $m_{\nu\nu^c}$ is fixed by the model to be of order 1 MeV. Thus this case is not realistic. However, one can consider the opposite case, case (c), where Λ_2 is much smaller than Λ_1 so that K_2 will be much smaller than both K_1 and $m_{\nu\nu^c}$.[5] In this case the approximate states and masses become

$$|\nu_1\rangle \simeq |\nu_L^c\rangle, \qquad \overline{M_1^H}(K_2 \ll K_1) \simeq K_1,$$

[5] K_2 may be comparable with m_S but still big enough to neglect the M_3^L term in (3,3) entry of the mass matrix in Eq. (5.9).

$$|\nu_2\rangle \simeq \frac{1}{\sqrt{\zeta^2+4A^2}}\left(\zeta|\nu_L\rangle + 2A|S_L^c\rangle\right), \qquad \overline{M_2^L}(K_2 \ll K_1) \simeq -\zeta^2 K_1,$$
$$|\nu_3\rangle \simeq \frac{1}{\sqrt{\zeta^2+4A^2}}\left(-2A|\nu_L\rangle + \zeta|S_L^c\rangle\right), \qquad \overline{M_3^L}(K_2 \ll K_1) \simeq -A^2 K_1 \qquad (5.11)$$

where $A \equiv \frac{K_2}{K_1}$ is used. From above one can define the mixing angle between active and sterile neutrinos as $\tan\theta \equiv \frac{2A}{\zeta}$.

Let us consider $\Lambda_1 \sim 10^{11}$ GeV, $\Lambda_2 \sim 1$ TeV and $m_S \sim 0.01$ eV (which does not affect the masses and the mixing angle much unless m_S is of the order $m_{\nu\nu^c}$). Then ζ and A are of the order of 10^{-7} and 10^{-8}, respectively. From Eq. (5.11), with the set of values taken for the parameters above, we have $\overline{M_1^H} = 10^4$ GeV, and ν_1 and ν_L^c decouple from the others. The masses for ν_2 and ν_3 are approximately 0.1 eV and 2×10^{-3} eV, respectively, with $\tan\theta \simeq 0.19$ ($\theta \simeq 10.6^o$, which is big enough to produce the active-sterile mixing required by the LSND data). Indeed, unlike the masses, the mixing angle is very sensitive to the exact value of K_2. The above values are for $K_2 = 2 \times 10^{-3}$ eV. Taking 10^{-3} eV instead would render the angle half as large. However, the mass for ν_3 becomes 2×10^{-3} eV while leaving $\overline{M_2^L}(K_2 \ll K_1)$ unchanged. The main point is that it is possible to have two light states having both active and sterile components with small mixing compatible with the solar, atmospheric and LSND neutrino experiments.

We comment on the case with $\Lambda_2 = 0$ as a limiting case of the above discussion. The important feature is that the coupling between ν_L^c and S_L^c disappears. This renders the above ν_3 state even lighter[6] and leaves the other states unchanged. However, the ν_2 and ν_3 states of Eq. (5.11) will be completely different. ν_3 will be almost a pure S_L^c state and decouples when m_S is considered negligible (and taken to be zero). The ν_2 and ν_1 states have a very small mixing, of the order of $\zeta \sim 10^{-7}$, between ν_L and ν_L^c. For the case where m_S is not negligible, we should take into account $|M_3^L|$, which is $\frac{2\lambda_1 v_3 m_{ee^c}}{m_{EE^c}}$ in the ALR model and $\frac{2m'_{\nu\nu^c} m'_{ee^c}}{m'_{EE^c}}$ in the Inert case. The only modification will be in ν_2 and ν_3 states and, in order to have masses smaller than 1 eV, M_3^L is allowed to be at most one order of magnitude bigger.[7] However, in this case the mixing between active-sterile flavor states would be too small ($\leq 3^o$) while the masses are ± 0.1 eV. Let us consider the case where

[6] As a matter of fact, it is massless unless the M_3^L term is included. So, negligible M_3^L with respect to m_S is considered below.

[7] m_S is assumed to be around 0.01 eV. As before, masses around 1 eV give physical masses also close to 1 eV and very large mixing.

M_3^L is at least two orders of magnitude smaller than m_S.[8] For $\Lambda_1 \sim 10^{11}$ GeV, the masses of ν_2 and ν_3 are 0.1 eV and 10^{-3} eV, respectively with $\tan\theta \simeq 0.1$ ($\theta \sim 5.7^o$). So, this case also yields a framework with two light states with almost fixed active-sterile mixing angle regardless of how small the Majorana mass of S_L^c is. However, the case with non-zero but small Λ_2 compared to Λ_1 yields a mixing very sensitive to the value of K_2 mainly due to the existence K_{12} coupling in the matrix. The possibility of having $\Lambda_2 = 0$ (or, in general, one of vanishing scale) has an advantage over the other cases discussed above as it may not always be possible to have two intermediate scale vev's for both of the singlet fields whose masses could be nonzero.

The discussion in this section can be generalized to three generations in a straightforward manner. However, one must be concerned about dangerous flavor changing neutral current interactions. Since the Higgs sectors of the models include three sets of Higgs bosons, one for each generation, the Glashow-Weinberg theorem [34] will be violated leading to tree-level flavor changing neutral currents mediated by neutral Higgs bosons. In addition, lepton universality will be broken due to mixing between leptons and the $SU(2)_{R(R')}$ gaugino. One can, of course, fine-tune the relevant couplings or make the relevant Higgs fields very heavy, but these solutions are unnatural. There are alternatives discussed in the literature [35, 36]. If one chooses a basis such that only one neutral Higgs field, say the third generation field, gets a vev, and one also considers a discrete symmetry which distinguishes between generations, then there will be no mediation of flavor-changing neutral currents between the first two generations. This can be achieved by assigning even parity for the third family Higgs fields and odd parity for those of the first two families. A classification of such generational symmetries has been done [36]. Unlike the quark sector, it is not possible to remove all flavor changing neutral interactions from the lepton sector within the above framework. Bounds on such interactions involving the tau sector are much weaker than those involves muon-electron interactions, and such models may be phenomenologically acceptable.

We note that the discussion in this study can also be carried out within the context of the Additional Neutral Fermion (ANF) method. This would have results similar to the HDO method. However, our main point is to show that, in the neutral lepton sectors of the ALR

[8] Indeed, it doesn't matter how small M_3^L is. The mixing angle is not sensitive to the M_3^L parameter and is very stable. M_3^L doesn't affect much the masses of ν_2 and ν_3 either. So S_L^c can be safely considered a pseudo-Dirac particle.

and Inert models, it is possible to have a framework in which sterile neutrinos exist naturally having small mixings with the active neutrinos, consistent with the LSND result. As shown above, the HDO method allows us to realize such a framework and so it is unnecessary to extend the discussion to the more complicated ANF method as well.

VI. CONCLUSIONS

It is possible that ongoing neutrino experiments, such as MiniBooNe, will make the necessity of one or more sterile neutrinos unavoidable. A feature of E_6 models is that the fundamental representation of the group contains a number of isosinglets that would be natural candidates for such neutrinos. It is important to analyze these models to see if the various neutral fermions can give rise to an acceptable phenomenology. In an earlier paper [19], we considered E_6 subgroups which contain an extra $SU(2)$ group, concentrating on the Alternative Left-Right (ALR) and Inert models, and we examined the neutrino spectrum in a non-supersymmetric framework.

In that paper [19], it was shown that both the ALR and Inert models predict neutrino sectors which are phenomenologically unacceptable. The lightest state always contained only isosinglets, and each generation contained isodoublet neutrino states with masses of the order of the $Q = 2/3$ quark mass. Three methods that alleviated these problems were discussed. The first was the Discrete Symmetry (DS) method, in which a discrete symmetry is imposed to eliminate the tree-level Dirac mass. Dirac neutrino masses can only be generated at one-loop, and the parameters can easily be adjusted to give masses in the correct mass range. However, there were still very light isosinglet masses, and no mixing with the isodoublets. The second method, the Higher Dimensional Operators (HDO) method, required additional Higgs fields and an intermediate scale. This method used dimension-5 operators to remove the very light isosinglets and thus the lightest neutrino states were isodoublets in the correct mass range. An interesting feature of this model was that the coupling of the isodoublet neutrinos to the W-boson is somewhat suppressed. Finally, the Additional Neutral Fermion (ANF) method required the existence of new particles as well as discrete symmetries and was able to accommodate mixing between the light isodoublet neutrinos and the sterile neutrinos.

In this paper, we have considered the supersymmetric version of the ALR and Inert

models. An attractive feature of the supersymmetrization of the models is that the Higgs fields can be taken to be supersymmetric partners of some of the exotic neutral fermions in the **27**-plet. The ALR and Inert model symmetries then constrain the allowed terms in the superpotential. If one assumes that R-parity is conserved, then one finds the mass matrix divides into two sectors. In the $R = +1$ sector, the active neutrino gets a large mass, of the order of the $Q = 2/3$ quark mass, and mixes maximally with the isosinglet ν_L^c. In the $R = -1$ sector, one finds two heavy states and one very light isosinglet state. Thus one has the same problems as in the non-supersymmetric case. However, there is an interesting alternative. It is possible to mix the two sectors through soft R-parity violating terms. Several cases were analyzed, and it was found that the mass and mixing problems still exist. The only change is that there can now be a small mixing between the active neutrino and one of the isosinglet neutrinos.

Thus, it was necessary to go beyond the minimal content of these models. In the Discrete Symmetry method, a discrete symmetry, which is generation-dependent, was used to eliminate the Dirac neutrino mass at tree-level. As in the non-supersymmetric picture, one-loop corrections can give a mass in the right mass range. Mixing with the isosinglet ν_L^c, however, remains maximal, and there is no substantial mixing with other isosinglet neutrinos. We next considered the Higher Dimensional Operators method, in which an intermediate scale is introduced, as well as some isosinglet Higgs fields from a $\overline{\mathbf{27}}$ representation of E_6. Various cases were considered, and it was shown that a fully acceptable model, with masses and mixing angles in the phenomenologically preferred region, can be obtained. We also briefly discussed the generation of tree-level flavor-changing neutral currents due to the proliferation of Higgs doublets in the models, and noted that the currents can be eliminated in the quark and $(e - \mu)$ sectors, but not entirely in the τ sector.

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APPENDIX

In this Appendix, we give the masses and the corresponding states for the case in which $\not{R}$-parity is included in the ALR model. We give the description for ALR but the results apply for Inert as well. The results are summarized in Table II. We would like to comment on the assumptions used to get these results. Recall that $M_3^L \equiv 2\lambda_1 v_3 m_{ee^c}/m_{EE^c}$.

TABLE II: The eigenstates and masses through $\not{R}$-parity for different hierarchies among m_S, M_3^L, and $m_{\nu\nu^c}$. Here ξ and Θ are defined as $\frac{m_S}{m_{\nu\nu^c}}$ and $\frac{M_3^L}{6m_{\nu\nu^c}}\left(2+\frac{\xi^2-2}{\sqrt{1+\xi^2}}\right)$, respectively.

Cases	States		Masses
$m_S \ll \lvert M_3^L\rvert \ll m_{\nu\nu^c}$	Light	$\lvert\nu_3\rangle \simeq \lvert S_L^c\rangle$	$M_3^L \simeq \frac{2\lambda_1 v_3 m_{ee^c}}{m_{EE^c}}$
	Heavy	$\lvert\nu_{1,2}\rangle \simeq \frac{1}{\sqrt{2}}\left(\lvert\nu_L\rangle \pm \lvert\nu_L^c\rangle\right)$	$\overline{M_{1,2}^H} \simeq \pm m_{\nu\nu^c}$
$\lvert M_3^L\rvert \ll m_S \ll m_{\nu\nu^c}$	Light	$\lvert\nu_3\rangle \simeq \lvert S_L^c\rangle - \xi\lvert\nu_L^c\rangle$	$\overline{M_3^L} \simeq \frac{2\lambda_1 v_3 m_{ee^c}}{m_{EE^c}}\left(1-\frac{2}{3}\xi^2\right)$
	Heavy	$\lvert\nu_{1,2}\rangle \simeq \frac{\left(\lvert\nu_L\rangle \pm \lvert\nu_L^c\rangle \pm \xi\lvert S_L^c\rangle\right)}{\sqrt{2}}$	$\overline{M_{1,2}^H} \simeq \pm m_{\nu\nu^c}\left(1+\frac{1}{2}\xi^2\right)$
$\lvert M_3^L\rvert \ll m_S \sim m_{\nu\nu^c}$	Light	$\lvert\nu_3\rangle \simeq \frac{-\xi\lvert\nu_L^c\rangle + \lvert S_L^c\rangle}{\sqrt{1+\xi^2}}$	$\overline{M_3^L} \simeq \frac{2\lambda_1 v_3 m_{ee^c}}{3m_{EE^c}}\left(1+\frac{2-\xi^2}{\sqrt{1+\xi^2}}\right)$
	Heavy	$\lvert\nu_{1,2}\rangle \simeq \frac{\left(\sqrt{1+\xi^2}\lvert\nu_L\rangle \pm \lvert\nu_L^c\rangle + \xi\lvert S_L^c\rangle\right)}{\sqrt{2(1+\xi^2)}}$	$\overline{M_{1,2}^H} \simeq m_{\nu\nu^c}\sqrt{1+\xi^2}\,(\pm 1+\Theta)$

For the case (i), $m_S \ll |M_3^L| \ll m_{\nu\nu^c}$, we keep only $O(M_3^L/m_{\nu\nu^c})$ and $O(m_S/M_3^L)$ terms but not terms $O(m_S/m_{\nu\nu^c})$ and $O(m_S/M_3^L)$. The next order correction to the masses and the states listed in Table II are of the order $O((m_S/M_3^L)^2)$ and $O(m_S/m_{\nu\nu^c})$ which are presumed very small and negligible. Due to the absence of $O(m_S/M_3^L)$ terms in mass eigenvalues, $\nu_{1,2}$ does not have a S_L^c component.

For the case (ii), $|M_3^L| \ll m_S \ll m_{\nu\nu^c}$, only $O(m_S/m_{\nu\nu^c})$ and $O(M_3^L/m_S)$ terms are kept, such that the orthogonality of $|\nu_1\rangle$ and $|\nu_2\rangle$ is satisfied up to the order of $(\frac{m_S}{m_{\nu\nu^c}})^2$. If $m_{\nu\nu^c}$ is allowed to have values less than eV, the physical neutrino states $\nu_{1,2}$ in the second row of

Table II would give a $\mathbf{3+1}$ structure. This requires extreme fine-tuning. Furthermore, we should note that the sterile state $|\nu_L^c\rangle + \frac{m_S}{m_{\nu\nu^c}}|S_L^c\rangle$ mixes maximally with $|\nu_L\rangle$, which would be inconsistent with the constraints from the LSND result. In this case S_L^c has a small mixing with ν_L^c. The lightest state ν_3 could have the desired light mass, but it is totally sterile and has no chance to be detected.

Finally, the case (iii), $|M_3^L| \ll m_S \sim m_{\nu\nu^c}$, which is possible when we consider fairly large soft-term couplings μ_A, is obviously a modification of the case (ii) when $m_{\nu\nu^c}$ and m_S are comparable. To get the results given in the third row of Table II, we neglected terms of orders $O(|M_3^L|/m_{\nu\nu^c})$ and $O(|M_3^L|/m_S)$. The sterile state is now composed of ν_L^c and S_L^c mixing almost maximally and the almost purely sterile state indeed has an active component whose mixing is proportional to $O(|M_3^L|/m_{\nu\nu^c})$. Their masses are also modified accordingly.

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